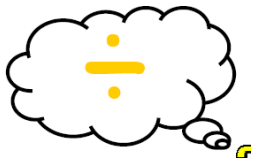
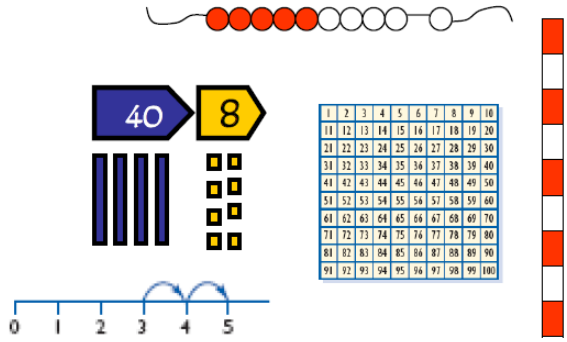
Progression in Teaching Division

Mental Skills



Recognise the size and position of numbers
Count back in different steps 2s, 5s, 10s
Halve numbers to 20
Recognise division as repeated subtraction
Quick recall of division facts
Use known facts to derive associated facts
Divide by 10, 100, 1000 and understanding the effect
Divide by multiples of 10

Models and Images



Counting apparatus
Arrays
100 squares
Number tracks
Numbered number lines
Marked but unnumbered lines
Empty number lines.
Multiplication squares
Models and Images charts
ITPs–Multiplication grid, Number Dials, Grouping, Remainders
Numicon

In developing a written method for division, it is important that children understand the concept of division, in that it is:

- **repeated subtraction**
- **sharing into equal amounts**

They also need to understand and work with certain principles, i.e. that it is:

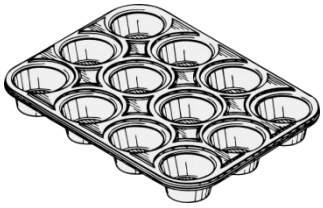
- **the inverse of multiplication**
- **not commutative i.e. 15 ÷3 is not the same as 3 ÷ 15**
- **not associative i.e. 30 ÷ (5 ÷ 2) is not the same as (30 ÷ 5) ÷ 2**

YR

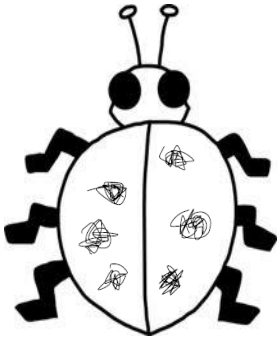
Early Learning Goal:
Children solve problems, including halving and sharing.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate sharing items or putting items into groups using items such as egg boxes, ice cube trays and baking tins which are arrays.



They may develop ways of recording calculations using pictures, etc.



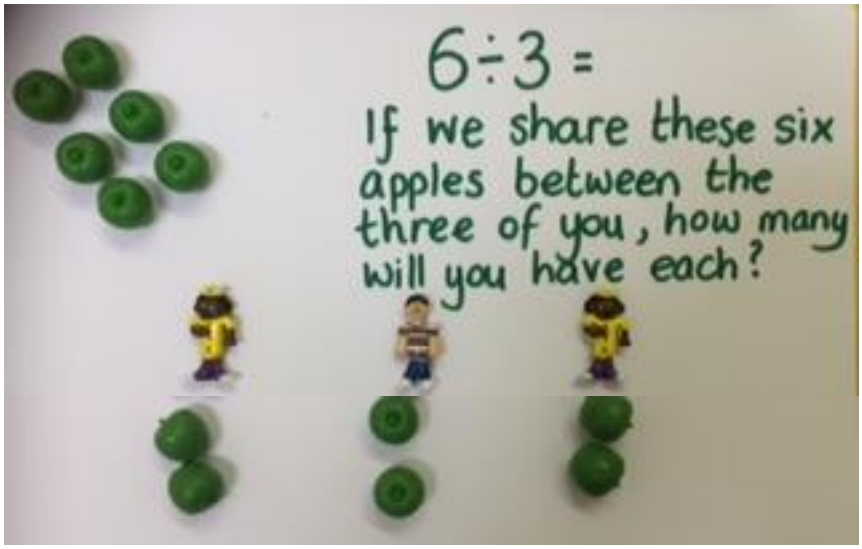
A child's jotting showing halving six spots between two sides of a ladybird.

A child's jotting showing how they shared the apples at snack time between two groups.

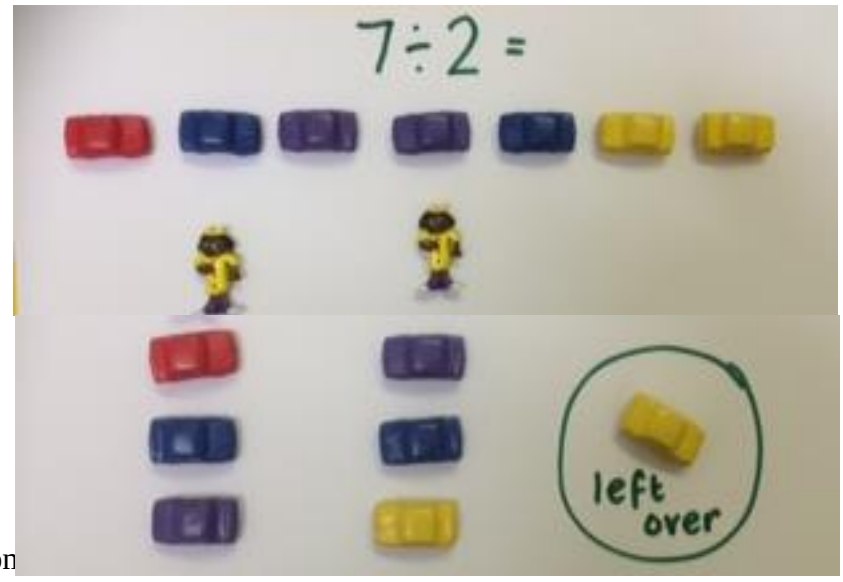
Y1

End of Year Objective:
Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

In year one, children will continue to solve division problems using practical equipment and jottings. They should use the equipment to share objects and separate them into groups, answering questions such as 'If we share these six apples between the three of you, how many will you each have? How do you know?' or 'If six football stickers are shared between two people, how many do they each get?' They may solve both of these types of question by using a 'one for you, one for me' strategy until all of the objects have been given out.



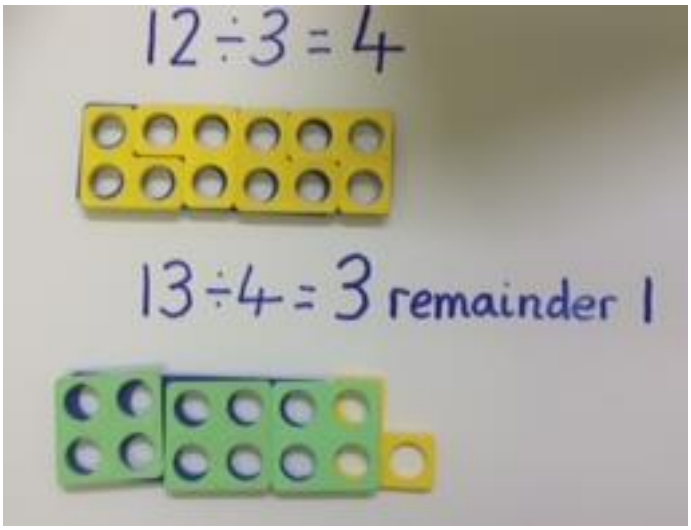
Children should be introduced to the concept of simple remainders in their calculations at this practical stage, being able to identify that the groups are not equal and should refer to the remainder as '… left over'.



Y2

End of Year Objective:
Calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs.

Children will utilise practical equipment to represent division calculations as grouping (repeated subtraction) and use pictoral jottings to support their calculation, e.g.



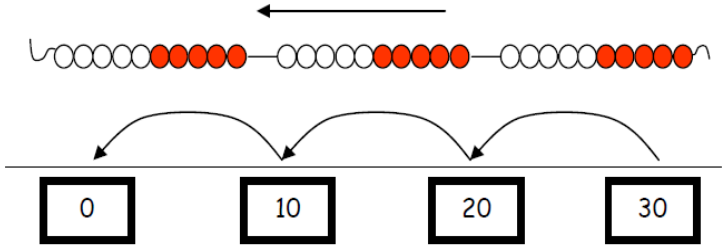
Children need to understand that this calculation reads as 'How many groups of 3 are there in 12?' They should also continue to develop their knowledge of division with remainders, e.g.
13 ÷ 4 = 3 remainder 1

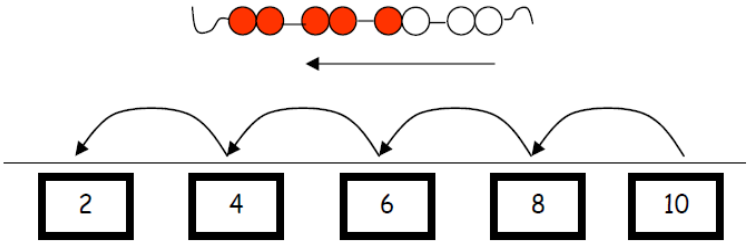
Children need to be able to make decisions about what to do with remainders after division and round up or down accordingly. In the calculation 13 ÷ 4, the answer is 3 remainder 1, but whether the answer should be rounded up to 4 or rounded down to 3 depends on the context, as in the examples below:

I have £13. Books are £4 each. How many can I buy?
Answer: 3 (the remaining £1 is not enough to buy another book)

Apples are packed into boxes of 4. There are 13 apples. How many boxes are needed?
Answer: 4 (the remaining 1 apple still needs to be placed into a box)

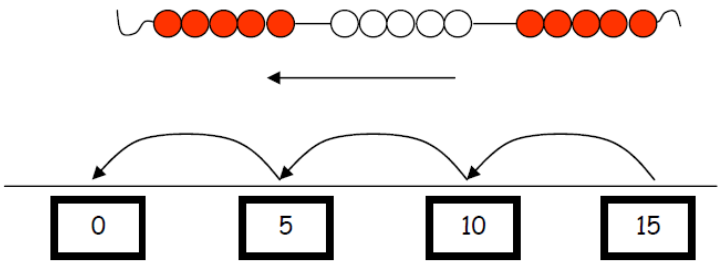
Count back in tens

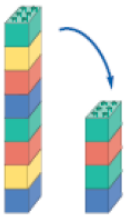




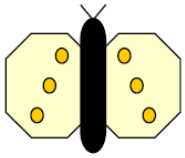
Count back in twos

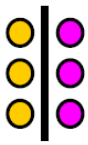
Count back in fives





half of 8 is 4
8 ÷ 2 = 4



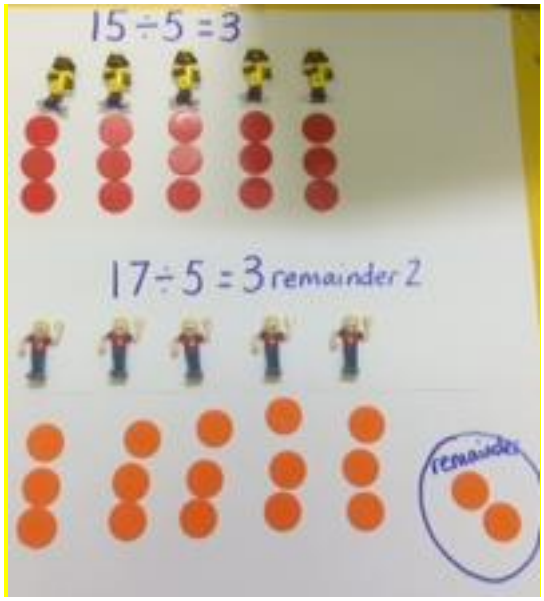


Half of 6 is 3
½ of 6 = 3

Know halves

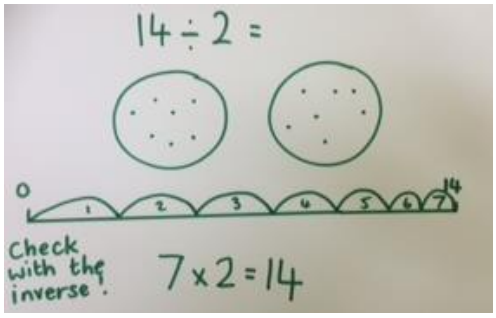
Use known multiplication facts to work out corresponding division facts

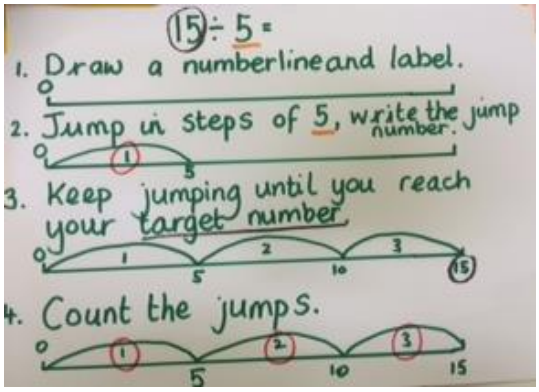
If 2 x 10 = 20
then
20 ÷ 10 = 2
20 ÷ 2 = 10



Children must understand division as sharing. Once secure they should be able to find simple remainders. Divisions given should be within the time tables for Year 2. Children should also record pictorially alongside the number statement.

When moving towards a numberline it is important for children to show both the numberline and the sharing alongside one another to allow them to make the connection.





Children should then use a numberline to work out number statements. They must be secure in sharing before moving on to a number line.

Y3

End of Year Objective:
Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, progressing to formal written methods.*

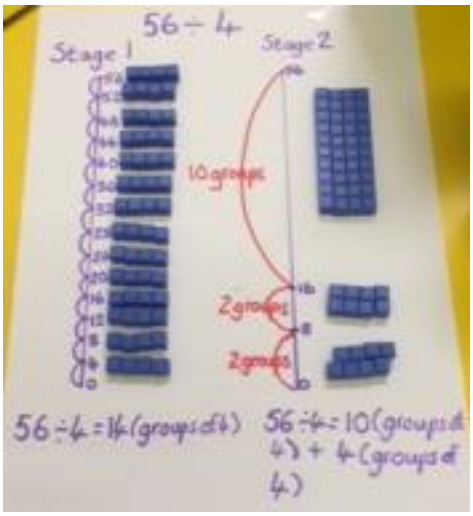
**Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage.*

Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4

It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

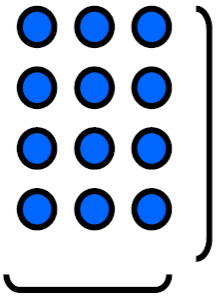
Initially, children will continue to use division by grouping (including those with remainders), where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10).

In preparation for developing the 'chunking' method of division, children should first use the repeated subtraction on a vertical number line alongside the continued use of practical equipment. There are two stages to this:



After each group has been subtracted, children should consider how many are left to enable them to identify the amount remaining on the number line.

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.



12 divided into groups of 3 gives 4 groups

$12 \div 3 = 4$

12 divided into groups of 4 gives 3 groups

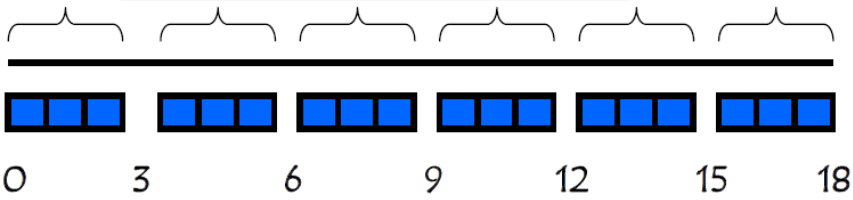
$12 \div 4 = 3$

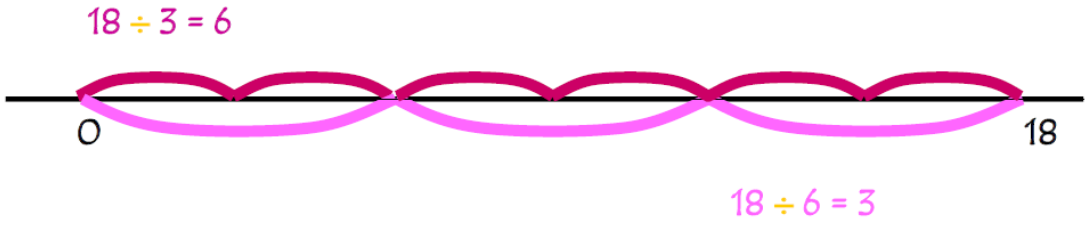
Reinforce division as grouping through the use of arrays

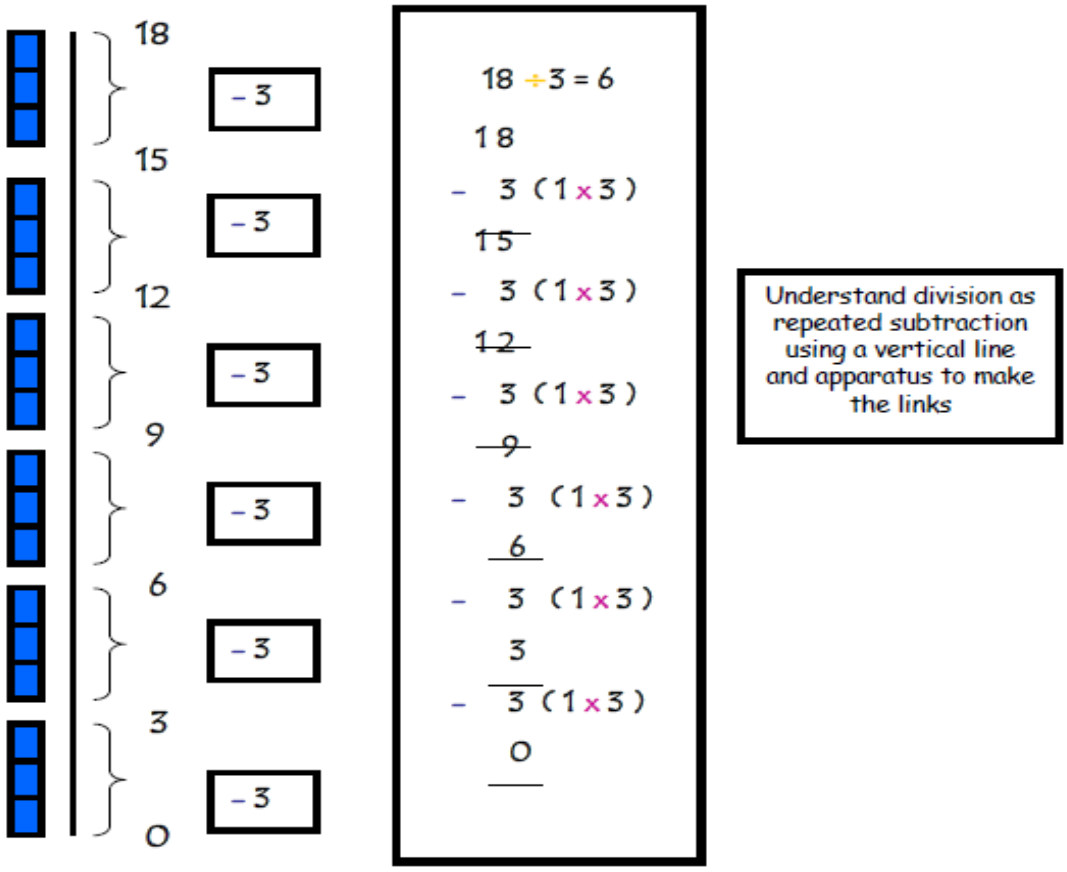
Represent 'groups' for division on a number line using apparatus alongside the line

18 divided into groups of 3

$18 \div 3 = 6$



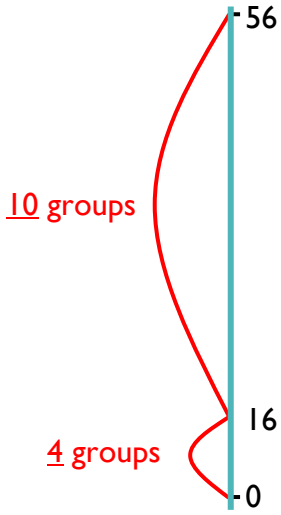




Y4

End of Year Objective:
Divide numbers up to 3 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Children will continue to develop their use of grouping (repeated subtraction) to be able to subtract multiples of the divisor, moving on to the use of the 'chunking' method.




```
   14
4 ) 56
  - 40      10x
   16
  - 16       4x
    0
```

Children should write their answer above the calculation to make it easy for them and the teacher to distinguish.

Answer: 14

The number line method used in year 3 can be linked to the chunking method to enable children to make links in their understanding.

When developing their understanding of 'chunking', children should utilise a 'key facts' box, as shown below. This enables an efficient recall of tables facts and will help them in identifying the largest group they can subtract in one chunk. Any remainders should be shown as integers, e.g.

73 ÷ 3

```
   24r1
3 ) 73
  - 30      10x
   43
  - 30      10x
   13
  -  6       2x
    7
  -  6       2x
     1
```

Key facts box

1x	3
2x	6
5x	15
10x	30

By the end of year 4, children should be able to use the chunking method to divide a three digit number by a single digit number. To make this method more efficient, the key facts in the menu box should be extended to include 4x and 20x, e.g.

196 ÷ 6

```
   32r4
6 ) 196
  - 120     20x
    76
  -  60     10x
    16
  -  12      2x
     4
```

Key facts box

1x	6
2x	12
4x	24
5x	30
10x	60
20x	120

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Children need to see that as the numbers get larger, large chunk subtraction is the more efficient method. Multiples of the divisor (large chunks) are taken away. Multiplication facts are needed to see the size of the 'chunk'.

What facts do I know about the 7 times-table?

100 ÷ 7 = 14 r 2

100
- 70 (10 × 7)
30
- 28 (4 × 7)
2

518 ÷ 7 = 74

518
- 350 (50 × 7)
168
- 140 (20 × 7)
28
- 28 (4 × 7)
0

Fact Box

1 × 7 = 7
2 × 7 = 14
5 × 7 = 35
10 × 7 = 70
20 × 7 = 140
50 × 7 = 350
100 × 7 = 700

Y5

End of Year Objective:
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Children may continue to use the key facts box for as long as they find it useful. Using their knowledge of linked tables facts, children should be encouraged to use higher multiples of the divisor. Any remainders should be shown as integers, e.g.

523 ÷ 8

```
   65r3
8 ) 523
  - 320   40x
    203
  - 160   20x
     43
  -  40    5x
      3
```

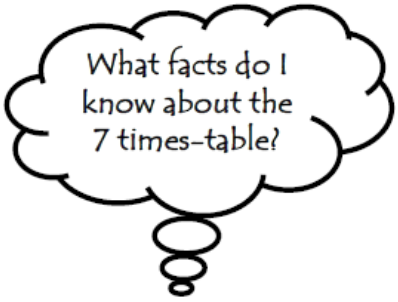
By the end of year 5, children should be able to use the chunking method to divide a four digit number by a single digit number. If children still need to use the key facts box, it can be extended to include 100x.

2458 ÷ 7

```
   351 r 1
7 ) 2458
  - 2100   300x
    358
  -  350    50x
     8
  -   7     1x
      1
```

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Children need to see that as the numbers get larger, large chunk subtraction is the more efficient method. Multiples of the divisor (large chunks) are taken away. Multiplication facts are needed to see the size of the 'chunk'.



100 ÷ 7 = 14 r 2

100
- 70 (10 x 7)
30
- 28 (4 x 7)
2

518 ÷ 7 = 74

518
- 350 (50 x 7)
168
- 140 (20 x 7)
28
- 28 (4 x 7)
0

Fact Box

1 x 7 = 7
2 x 7 = 14
5 x 7 = 35
10 x 7 = 70
20 x 7 = 140
50 x 7 = 350
100 x 7 = 700

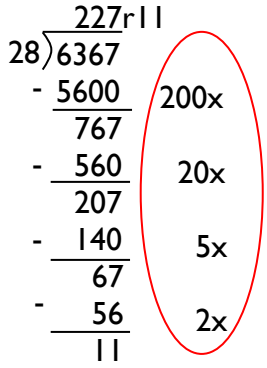
Y6

End of Year Objective:
Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.

Use written division methods in cases where the answer has up to two decimal places.

To develop the chunking method further, it should be extended to include dividing a four-digit number by a two-digit number, e.g.

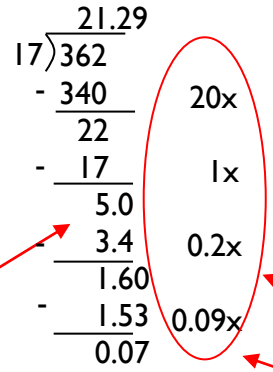
6367 ÷ 28



Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

In addition, children should also be able to use the chunking method and solve calculations interpreting the remainder as a decimal up to two decimal places, e.g. 362 ÷ 17

362 ÷ 17



To enable children to continue the calculation, they need to understand that 5 is the same as 5.0

When recalling and deriving multiplication and division facts, children should also identify decimal equivalents of times tables,
e.g. if 2 x 17 = 34, I know that
0.2 x 17 = 3.4
if 9 x 17 = 153, 0.9 x 17 = 15.3
so 0.09 x 17 = 1.53

For simple fraction and decimal equivalents, this could also be demonstrated using a simple calculation such as 13 ÷ 4 to show the remainder initially as a fraction.

Using practical equipment, children can see that for 13 ÷ 4, the answer is 3 remainder 1, or put another way, there are three whole groups and a remainder of 1. This remainder is one part towards a full group of 4, so is $\frac{1}{4}$. To show the remainder as a fraction, it becomes the numerator where the denominator is the divisor (the number that you are dividing by in the calculation).

3574 ÷ 8

$$
\begin{array}{rll}
8\overline{)3574} & & \\
-\ 3200 & 400\text{x} & \\
\hline
374 & & \\
-\ 320 & 40\text{x} & \\
\hline
54 & & \\
-\ 48 & 6\text{x} & \\
\hline
6 & &
\end{array}
$$

6 ← remainder

8 ← divisor

So 3574 ÷ 8 is $446\frac{6}{8}$

(when the remainder is shown as a fraction)

To show the remainder as a decimal relies upon children's knowledge of decimal fraction equivalents. For decimals with no more than 2 decimal places, they should be able to identify:

Half: $\frac{1}{2} = 0.5$

Quarters: $\frac{1}{4} = 0.25$, $\frac{3}{4} = 0.75$

Fifths: $\frac{1}{5} = 0.2$, $\frac{2}{5} = 0.4$, $\frac{3}{5} = 0.6$, $\frac{4}{5} = 0.8$

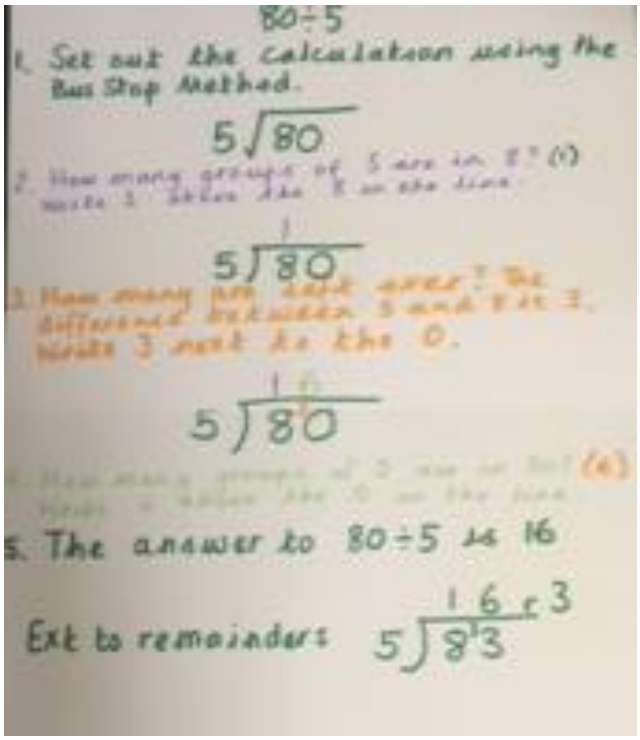
Tenths: $\frac{1}{10} = 0.1$, $\frac{2}{10} = 0.2$, $\frac{3}{10} = 0.3$, $\frac{4}{10} = 0.4$, $\frac{5}{10} = 0.5$, $\frac{6}{10} = 0.6$, $\frac{7}{10} = 0.7$, $\frac{8}{10} = 0.8$, $\frac{9}{10} = 0.9$

and reduce other equivalent fractions to their lowest terms.

In the example above, 3574 ÷ 8, children should be able to identify that the remainder as a fraction of $\frac{6}{8}$ can be written as $\frac{3}{4}$ in its lowest terms. As $\frac{3}{4}$ is equivalent to 0.75, the answer can therefore be written as 446.75

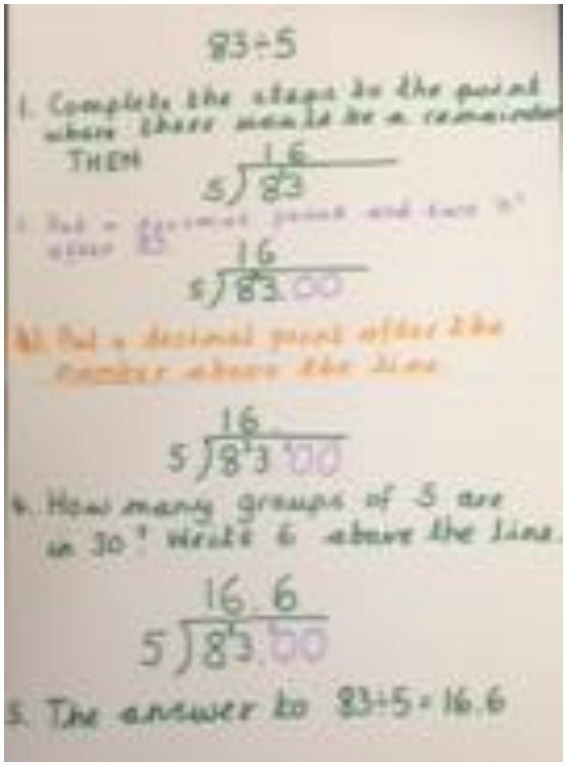
Once these methods are secure the formal long division method can be taught. Place value must be secure and children must know by heart the facts for the tables up to 12 x 12.

Long Division

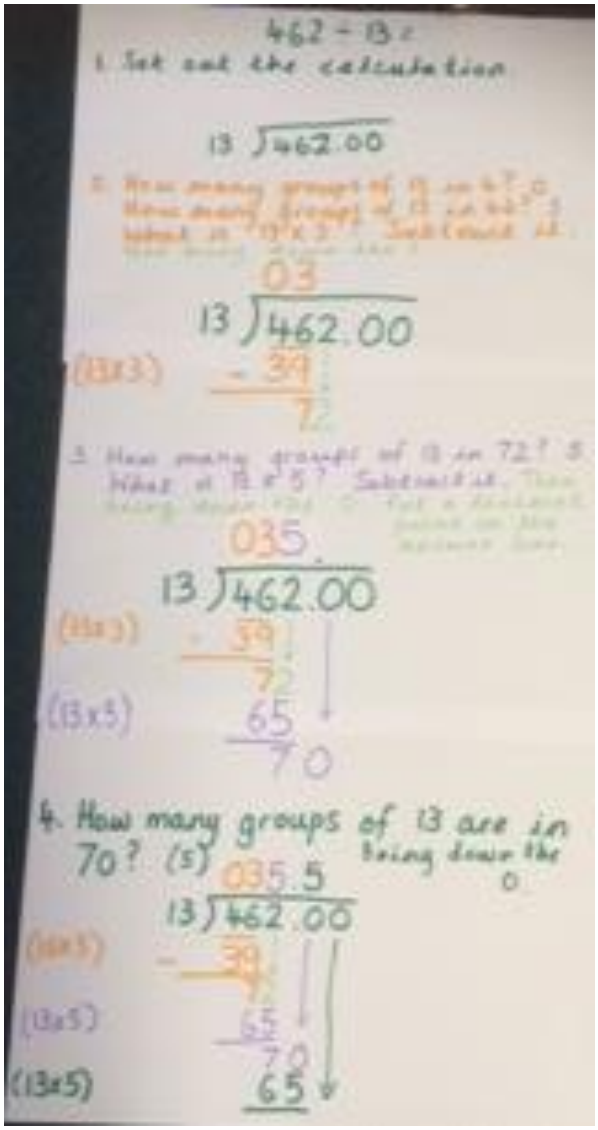


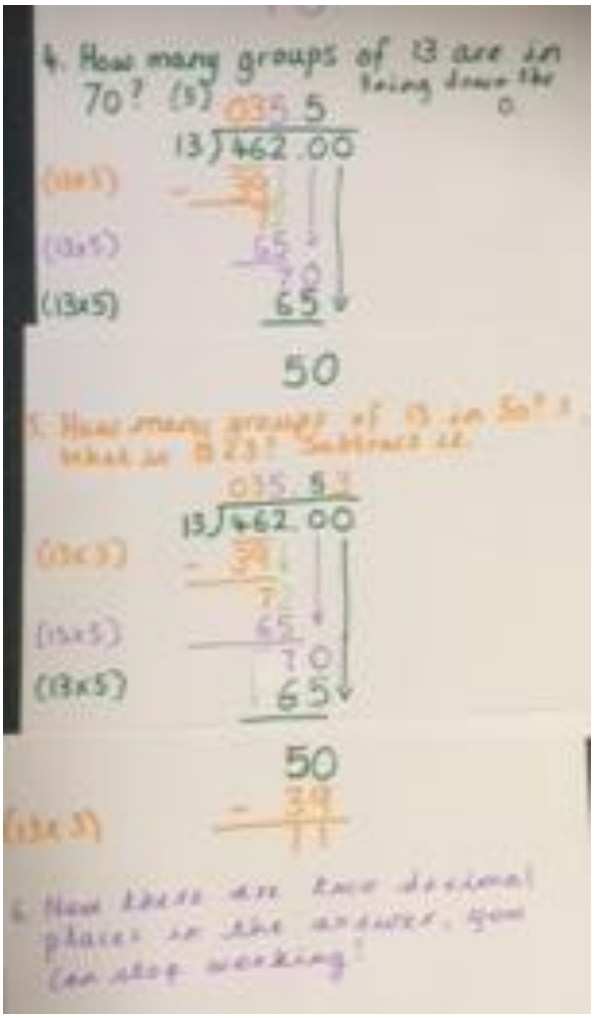
Step 1 - Use the Bus Stop Method to divide a 2-digit number by a single digit number with no remainder.

Step 2 - Use the Bus Stop Method to divide a 2-digit number with a remainder



Step 3 – Use the Bus Stop Method to divide a 3-digit number by a 2-digit number with a decimal answer.




4. How many groups of 13 are in 70? (5)
13)462.00
(13x5)
(13x5)
(13x5)
65
50
13)462.00
(13x3)
(13x5)
(13x5)
65
50
(13x3)